

KEK-TH-425
KEK Preprint 94-171
December 1994

Polarization of τ lepton from scalar tau decay as a probe of neutralino mixing

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Abstract

The τ lepton arising from the scalar tau ($\tilde{\tau}$) decay is naturally polarized. P_τ depends on the left-right mixing of the $\tilde{\tau}$ and the gaugino-higgsino mixing of the neutralino. The polarization P_τ could be measured from the energy distribution of the decay products of τ at future e^+e^- colliders. A measurement of P_τ and of the $\tilde{\tau}$ production cross section allows to determine both these mixing angles.

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1) Introduction

The Minimal Supersymmetric Standard Model (MSSM)[1] is one of the most promising candidates of the models beyond the standard model (SM)[2]. It predicts the existence of superpartners of SM particles below a few TeV to remove quadratic divergences which appear in radiative corrections of the SM; thus the model is free from the so-called hierarchy problem[3] of GUT models. It should be noted that the gauge couplings unify very precisely at high energy scale in MSSM [4], consistent with SUSY GUT predictions.

If supersymmetry is unbroken, a particle and its superpartner have equal mass. Since no superpartner has been found so far the symmetry must be broken rather badly. Several mechanisms to break supersymmetry have been proposed in the framework of supergravity and superstring models [5]. These models predict relations between various soft SUSY breaking parameters at the GUT scale (or Planck scale); these are the soft scalar masses m_i , gaugino masses M_i , and trilinear couplings A_i . The masses of SUSY particles at the weak scale are obtained by evolving the GUT scale parameters using the renormalization group equation (RGE) of the MSSM. Thus the observation of superpartners at e^+e^- or hadron colliders should give important information on the models that describe the origin of supersymmetry breaking.

In this paper, we study the production and decay of the superpartner of the tau ($\tilde{\tau}$) at future e^+e^- colliders in the framework of the MSSM. Production and decay of $\tilde{e}_{L,R}$ and $\tilde{\mu}_{L,R}$ at such colliders have already been discussed in various articles [6]. $\tilde{e}_{L,R}$ ($\tilde{\mu}_{L,R}$) is produced at e^+e^- colliders by s channel exchange of photon and Z^0 for $\tilde{\mu}_{L,R}$, and also by t channel exchange of neutralinos for $\tilde{e}_{L,R}$. The produced $\tilde{e}_{L,R}$ ($\tilde{\mu}_{L,R}$) then decays into the lightest neutralino χ_1^0 , which is a mixture of gauginos and higgsinos, plus an e (μ) through the gaugino component of the neutralino, giving a signal with leptons and missing momentum.

$\tilde{\tau}$ production and decay is different from that of \tilde{e} and $\tilde{\mu}$ because the τ lepton has a non-negligible Yukawa coupling. Due to this coupling, $\tilde{\tau}_R$ and $\tilde{\tau}_L$ mix, thus the mass eigenstates are not necessarily current eigenstates [7]. The same Yukawa coupling appears as a non-negligible $\tau\tilde{\tau}\tilde{H}_1^0$ coupling, where \tilde{H}_1^0 is a neutral higgsino[8]. This interaction is involved in $\tilde{\tau}$ decay processes, since the χ 's are mixtures of higgsinos and gauginos. Another feature of $\tilde{\tau}$ decay that distinguishes it from other slepton decays is that the τ lepton arising from the decay $\tilde{\tau} \rightarrow \chi\tau$ decays further in the detector, which enables us to measure the average polarization of the τ (P_τ) [9, 10]. The purpose of this paper is to point out that one can measure the $\tilde{\tau}_L - \tilde{\tau}_R$ mixing of $\tilde{\tau}_1$ and the higgsino-gaugino mixing of χ_i^0 by measuring both the cross section for $\tilde{\tau}$ production and the average polarization of τ from $\tilde{\tau}$ decays.

The rest of this paper is organized as follows. In section 2 we discuss $\tilde{\tau}_1\tilde{\tau}_1^*$ production at an e^+e^- collider with $\sqrt{s} = 500$ GeV. We point out that the cross section for $\tilde{\tau}_1$ production depends on $m_{\tilde{\tau}_1}$ and on the $\tilde{\tau}_1$ left-right mixing angle θ_τ . If a polarized electron beam is

available, θ_τ can be determined from the cross section, assuming $m_{\tilde{\tau}_1}$ is known (e.g. from kinematical distributions).

In section 3 we show that the τ lepton arising from $\tilde{\tau}_1$ decay is naturally polarized. The $P_\tau(\tilde{\tau}_1 \rightarrow \chi_i^0 \tau)$ depends on both θ_τ and the neutralino mixing. This latter dependence arises because the interaction of gauginos with (s)fermions preserves chirality, while the interaction of higgsinos flips it. Removing the θ_τ dependence by using the measured value of θ_τ , one can isolate information on the neutralino mixing. We also show that the branching ratio of the decay mode $\tilde{\tau}_1 \rightarrow \chi_1^0 \tau$, which provides a good signal for $\tilde{\tau}_1$ production, is sizable in a wide region of the parameter space.

P_τ can be measured through the energy distribution of the decay products of the polarized tau lepton. A complication arises since this distribution also depends on the energy distribution of the parent tau coming from $\tilde{\tau}_1$ decay. In section 4 we describe the dependence of the energy distribution on $m_{\tilde{\tau}}$, $m_{\chi_1^0}$ and P_τ . Possible backgrounds are also discussed here.

Section 5 is devoted to a discussion and summary. We also describe the predictions for the mass spectrum in the minimal supergravity model, and its implications.

2) The cross section for $\tilde{\tau}_1 \tilde{\tau}_1^*$ production

The scalar tau lepton $\tilde{\tau}_{L(R)}$ is the superpartner of $\tau_{L(R)}$. $\tilde{\tau}_R$ is an SU(2) singlet and has hypercharge -1 , while $(\tilde{\nu}_\tau, \tilde{\tau})_L$ is a SU(2) doublet and has hypercharge $-1/2$. The mass matrix of $\tilde{\tau}$ can be written as follows [7, 11],

$$\mathcal{M}_{\tilde{\tau}}^2 = \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{pmatrix} = \begin{pmatrix} m_L^2 + m_\tau^2 + 0.27D & -m_\tau(A_\tau + \mu \tan \beta) \\ -m_\tau(A_\tau + \mu \tan \beta) & m_R^2 + m_\tau^2 + 0.23D \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix} \quad (1)$$

where m_R and m_L are soft breaking scalar mass parameters of $\tilde{\tau}_R$ and $(\tilde{\nu}_\tau, \tilde{\tau})_L$, A_τ is a trilinear coupling of $\tilde{\tau}_L \tilde{\tau}_R H_1$, and μ is the Higgsino mass parameter. $\tan \beta = \langle H_1^0 \rangle / \langle H_2^0 \rangle$ is the ratio of the vacuum expectation values of the two Higgs doublets of the MSSM, and $D = -m_Z^2 \cos(2\beta)$. $\tilde{\tau}_R$ and $\tilde{\tau}_L$ then mix to form two mass eigenstates $\tilde{\tau}_1$ and $\tilde{\tau}_2$ ($m_{\tilde{\tau}_1} < m_{\tilde{\tau}_2}$):

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_\tau & \sin \theta_\tau \\ -\sin \theta_\tau & \cos \theta_\tau \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}. \quad (2)$$

At future e^+e^- linear colliders such as JLC, it is proposed to have a highly polarized electron beam[12]. At the Stanford Linear Collider (SLC), an average polarization of 80% has already been achieved in the 1994 run. At future linear colliders, a beam polarization of more than 90% is expected[13, 14]. Having a polarized beam also helps to reduce background from W^+W^- pair

production, which makes SUSY analyses considerably easier[14]. Thus we give the differential and total cross sections ($d\sigma/d \cos \theta_\tau, \sigma$) for $\tilde{\tau}_1$ production by e^+e^- annihilation after taking spin sum and average only for the positron. $\tilde{\tau}_1\tilde{\tau}_1^*$ production occurs through s-channel exchange of γ and Z^0 :

$$\frac{d\sigma}{d \cos \theta}(e^+e^- \rightarrow \tilde{\tau}_1\tilde{\tau}_1^*) = \frac{\beta_f}{32\pi s} \frac{1}{2} |T(e^+e^- \rightarrow \tilde{\tau}_1\tilde{\tau}_1^*)|^2 \sin^2 \theta; \quad (3a)$$

$$\sigma(e^+e^- \rightarrow \tilde{\tau}_1\tilde{\tau}_1^*) = \frac{\beta_f}{3 \cdot 16\pi s} \frac{1}{2} |T(e^+e^- \rightarrow \tilde{\tau}_1\tilde{\tau}_1^*)|^2; \quad (3b)$$

where

$$T(e^+e^- \rightarrow \tilde{\tau}_1\tilde{\tau}_1^*) = \sqrt{2}s\beta_f \left[-g_Z^2 [\cos^2 \theta_\tau (\frac{1}{2} + \sin^2 \theta_W) + \sin^2 \theta_W \sin^2 \theta_\tau] \right. \\ \cdot \frac{1}{s - m_Z^2 + i\Gamma_Z m_Z} \times \begin{pmatrix} \sin^2 \theta_W \\ -\frac{1}{2} + \sin^2 \theta_W \end{pmatrix} \begin{array}{l} h_e = 1/2 \\ h_e = -1/2 \end{array} \\ \left. - e^2 \frac{1}{s} \right]. \quad (4)$$

Here h_e is the helicity of the initial electron, and $\beta_f = \sqrt{1 - 4m_{\tilde{\tau}_1}^2/s}$.^{*}

In the limit $s \gg m_Z$, Eq.(3) reduces to

$$T(h_e = 1/2) = -\sqrt{2}\beta_f g^2 \tan^2 \theta_W \left(\frac{1}{2} \cos^2 \theta_\tau + \sin^2 \theta_\tau \right); \quad (5a)$$

$$T(h_e = -1/2) = -\frac{\sqrt{2}\beta_f g^2}{4} \left[(1 + \tan^2 \theta_W) \cos^2 \theta_\tau + 2 \tan^2 \theta_W \sin^2 \theta_\tau \right]. \quad (5b)$$

If $h_e = 1/2$ (or $P_e = 1$), only $U(1)$ gauge boson exchange survives, thus the cross section for $\tilde{\tau}_R\tilde{\tau}_R^*$ production ($\cos \theta_\tau = 0$) is 4 times larger than that for $\tilde{\tau}_L\tilde{\tau}_L^*$ ($\cos \theta_\tau = 1$) as one can see from Eq.(5a) and (3b)[14]. On the other hand, due to the contribution of the exchange of the $SU(2)$ gauge boson, the cross section for $\tilde{\tau}_L\tilde{\tau}_L^*$ production is larger than that for $\tilde{\tau}_R\tilde{\tau}_R^*$ if $h_e = -1/2$.

In fig.1, we show the θ_τ dependence of the cross section for $\tilde{\tau}_1\tilde{\tau}_1^*$ production. We take $\sqrt{s} = 500$ GeV and $m_{\tilde{\tau}_1} = 150$ GeV. The solid curve is the cross section for $P_e = 1$, and the dashed curve is for $P_e = 0$. If the electron beam is unpolarized, the cross section is about 0.06 pb and has very mild dependence on θ_τ . Thus it is very difficult to distinguish the value of θ_τ from the unpolarized cross section only.

^{*}For the expression for $\tilde{\tau}_2\tilde{\tau}_2^*$ production, replace $\cos \theta_\tau$ and $\sin \theta_\tau$ by $-\sin \theta_\tau$ and $\cos \theta_\tau$.

On the other hand, for a right-handed electron beam $\sigma_{\tilde{\tau}_R} \simeq 4\sigma_{\tilde{\tau}_L}$ as expected. In this case the minimum cross section for $\tilde{\tau}_1\tilde{\tau}_1^*$ production is about 0.02 pb for the given choice of parameters. $\mathcal{L} = 30 \text{ fb}^{-1}$ might be available per year at future e^+e^- colliders [12], which means at least six hundred $\tilde{\tau}$ events can be used for the analysis of the nature of $\tilde{\tau}_1$. If one knows $m_{\tilde{\tau}_1}$ very precisely, e.g. from a threshold scan, one may extract θ_τ from the measurement of the production cross section. Assuming all $\tilde{\tau}$ production events are identified, the cross section can be measured with statistical error of less than 4% for $m_{\tilde{\tau}_1} = 150 \text{ GeV}$. This corresponds to less than $\pm 9^\circ$ of uncertainty for θ_τ . Notice that the soft breaking mass parameters m_{LL}^2 , m_{RR}^2 , m_{LR}^2 which appeared in Eq.(1) could be calculated if one further measures $m_{\tilde{\tau}_2}$; these measurements give us a unique opportunity to extract the trilinear coupling A_τ in m_{LR}^2 .

How can $\tilde{\tau}_1$ production be identified? The lighter $\tilde{\tau}_1$ decays into τ and a neutralino (χ_i^0) or ν_τ and a chargino (χ_i^-). Here, χ_i^0 is a mixture of \tilde{B} , \tilde{W}_3 , \tilde{H}_1 and \tilde{H}_2 , while χ_i^- is a mixture of \tilde{W} and \tilde{H} . The mass matrices of the neutralinos (χ_i^0) and charginos (χ_i^\pm) are given as follows[1]:

$$\begin{aligned} \mathcal{M}_N(\tilde{B}, \tilde{W}_3, \tilde{H}_1, \tilde{H}_2) &= \\ \left(\begin{array}{cccc} M_1 & 0 & -m_Z \sin\theta_W \cos\beta & m_Z \sin\theta_W \sin\beta \\ 0 & M_2 & m_Z \cos\theta_W \cos\beta & -m_Z \cos\theta_W \sin\beta \\ -m_Z \sin\theta_W \cos\beta & m_Z \cos\theta_W \cos\beta & 0 & -\mu \\ m_Z \sin\theta_W \sin\beta & -m_Z \cos\theta_W \sin\beta & -\mu & 0 \end{array} \right), \end{aligned} \quad (6a)$$

$$\mathcal{M}_c(\tilde{W}, \tilde{H}) = \left(\begin{array}{cc} M_2 & m_W \sqrt{2} \sin\beta \\ m_W \sqrt{2} \cos\beta & \mu \end{array} \right). \quad (6b)$$

These mass matrices are diagonalized by a real orthogonal matrix N for \mathcal{M}_N , and unitary matrices U and V for \mathcal{M}_c as follows:

$$U^* \mathcal{M}_C V^{-1} = M_D^c, \quad N \mathcal{M}_N N^{-1} = M_D^N. \quad (7)$$

Unlike the notation of Haber and Kane[1], we assume N to be real so that $m_{\chi_i^0}$ can be either positive or negative; the sign must be kept to understand later expressions. We take $|m_{\chi_1^0}| \leq |m_{\chi_2^0}| \leq |m_{\chi_3^0}| \leq |m_{\chi_4^0}|$ and $0 \leq m_{\chi_1^-} \leq m_{\chi_2^-}$. We assume the mass relation $M_1 = (5/3) \cdot \tan^2 \theta_W M_2$ for numerical calculations in order to reduce the number of parameters.

In the MSSM, there is a multiplicatively conserved R parity, which is assigned -1 for superparticles (superpartners of fermions, Higgs bosons and gauge bosons), and $+1$ otherwise. Therefore the decay of a superparticle creates at least one superparticle, so that the lightest of them is stable. Because no such particle has been found in cosmic rays, the lightest supersymmetric particle (LSP) must be neutral and weakly interacting. The lightest neutralino χ_1^0

is generally considered to be the best candidate for the LSP; it escapes detection in collider experiments because it is neutral and does not decay inside detectors.

If both of the $\tilde{\tau}_1^{(*)}$ decay into χ_1^0 and $\tau(\bar{\tau})$, the event should look like $\tau\bar{\tau} + \text{missing momentum}$, as the two χ_1^0 escape from detection. This is the simplest event signature for $\tilde{\tau}_1\tilde{\tau}_1^*$ production. If $\tilde{\tau}_1$ is the second lightest SUSY particle, all $\tilde{\tau}_1$ decay through this mode. The tau leptons can be identified through their decays into leptons or mesons, which we discuss in section 4.

If $\chi_i^0\tau$ or $\chi_i^-\nu$ modes are also kinematically accessible, the signatures for $\tilde{\tau}_1$ production are more complicated. The branching ratios into heavier neutralinos and charginos depend on θ_τ , M , $\tan\beta$, and μ very strongly, and could be dominant, as will be discussed in section 3. If we assume $\tilde{\tau}_1$ is the lightest of all sfermions (which means χ 's cannot decay into sfermions), those χ_i^0 and χ_i^- would further decay as

$$\begin{aligned}\chi_i^0 &\rightarrow \chi_{i'}^0 f\bar{f}, \chi_{i'}^\pm f\bar{f}', \\ \chi_i^\pm &\rightarrow \chi_{i'}^0 f\bar{f}', \chi_{i'}^\pm f\bar{f}.\end{aligned}\tag{8}$$

When the decay cascade is terminated, the final state contains one χ_1^0 due to the R parity conservation, again giving missing momentum to the event. When both $\tilde{\tau}_1$ and $\tilde{\tau}_1^*$ decay into a neutralino and a tau, we get an event $\tau\bar{\tau} + (\text{jets and/or leptons}) + \text{missing momentum}$. On the other hand, if both of $\tilde{\tau}_1$ decay into a chargino and ν_τ , the event does not contain any τ lepton in the final state (unless they come from -ino decays), so it is not a good $\tilde{\tau}_1$ signature.

One can in principle measure both $m_{\tilde{\tau}_1}$ and θ_τ by comparing the cross section for $\tilde{\tau}_1$ production at $P_e = 1$ and $P_e = 0$, as $\sigma(P_e = 0)$ is quite insensitive to θ_τ . However, the events at $P_e = 0$ suffer a large background from W^+W^- production where both W^+ and W^- decay into $\tau\nu$. The production cross section of W^+W^- is about 7 pb for $P_e = 0$, while it is only 0.05 pb for $P_e = 1$ as a right handed electron does not couple to transverse W bosons [14, 15]. Notice that most W bosons will be emitted in the forward direction when $P_e = 0$, while the signal tau leptons tend to be emitted at large angles as one can see from Eq.(3a). The cross section for W^+W^- production at $P_e = 0$ is reduced to 1 pb if we require $|\cos\theta| < 0.8$ for the azimuthal angle of the W bosons [12]. Then the background to the process $\tilde{\tau}_1\tilde{\tau}_1^* \rightarrow \tau\tau + \text{missing momentum}$ with a similar angle cut on the tau is roughly 10 fb and 0.5fb for $P_e = 0$ and $P_e = 1$, respectively. Although an accoplanarity angle cut reduces the background from W^+W^- even further, the measurement of $\sigma_{\tilde{\tau}_1}(P_e = 0)$ is clearly more difficult than that of $\sigma_{\tilde{\tau}_1}(P_e = 1)$.

3) τ polarization in $\tilde{\tau}_1$ decay

a) $P_\tau(\tilde{\tau}_1 \rightarrow \tau \chi_i^0)$

The τ lepton arising from the decay $\tilde{\tau}_1 \rightarrow \tau \chi_i^0$ further decays into $A \nu_\tau$ where $A = e\nu, \pi, \rho, a_1, \dots$. The decay distributions of the τ decay products depend on the polarization of the parent[9]. In particular, for each decay channel the momentum distribution of the decay products ($\pi^-, \rho^- \rightarrow \pi^-\pi^0, \dots$) differs significantly according to whether the parent is $\tau_R^-(h=1/2)$ or $\tau_L^-(h=-1/2)$. If the τ lepton is relativistic, P_τ can then be determined from the energy distribution of the decay products [10]. Compared to the case of \tilde{e} and $\tilde{\mu}$ decays, where the polarization of the final state lepton is not measurable, we therefore have one more piece of information available for the study of the nature of $\tilde{\tau}_1$ and χ_i^0 . In this section, we discuss the decay width of $\tilde{\tau}$ into $\chi_i^0 \tau$ and $\chi_i^- \nu_\tau$, while keeping our eyes on the polarization of the τ lepton.

The stau-ino-fermion interaction relevant for $\tilde{\tau}_1$ decay is expressed by the following Lagrangian[8]:

$$\mathcal{L} = \sum_{\substack{i=1,2 \\ j=1,\dots,4}} \tilde{\tau}_i \bar{\tau} (P_L a_{ij}^R + P_R a_{ij}^L) \chi_j^0 + \sum_{\substack{i=1,2 \\ j=1,2}} \tilde{\tau}_i \bar{\nu}_\tau P_R b_{ij} \chi_j^+, \quad (9)$$

where

$$\begin{pmatrix} a_{1j}^{R(L)} \\ a_{2j}^{R(L)} \end{pmatrix} = \begin{pmatrix} \cos \theta_\tau & \sin \theta_\tau \\ -\sin \theta_\tau & \cos \theta_\tau \end{pmatrix} \begin{pmatrix} a_{Lj}^{R(L)} \\ a_{Rj}^{R(L)} \end{pmatrix},$$

$$\begin{pmatrix} b_{1j} \\ b_{2j} \end{pmatrix} = \begin{pmatrix} \cos \theta_\tau & \sin \theta_\tau \\ -\sin \theta_\tau & \cos \theta_\tau \end{pmatrix} \begin{pmatrix} b_{Lj} \\ b_{Rj} \end{pmatrix}, \quad (10a)$$

$$a_{Lj}^R = -\frac{gm_\tau}{\sqrt{2}m_W \cos \beta} N_{j3}, \quad a_{Lj}^L = \frac{g}{\sqrt{2}} [N_{j2} + N_{j1} \tan \theta_W],$$

$$a_{Rj}^R = -\frac{2g}{\sqrt{2}} N_{j1} \tan \theta_W, \quad a_{Rj}^L = -\frac{gm_\tau}{\sqrt{2}m_W \cos \beta} N_{j3},$$

$$b_{Lj} = -g U_{j1}, \quad b_{Rj} = \frac{gm_\tau}{\sqrt{2}m_W \cos \beta} U_{j2}. \quad (10b)$$

The coefficients of the terms which flip chirality of the (s)fermion (a_{Ri}^L, a_{Li}^R) are proportional to the τ Yukawa coupling $gm_\tau/(m_W \cos \beta \sqrt{2})$. Notice that this coupling is proportional to $\cos^{-1} \beta$, which is close to $\tan \beta$ when $\tan \beta \gg 1$.

The decay width of the process $\tilde{\tau}_1 \rightarrow \chi_i^0 \tau$ is expressed as

$$\Gamma(\tilde{\tau}_1 \rightarrow \chi_i^0 \tau) = \frac{\beta_f}{16\pi m_{\tilde{\tau}}} |\mathcal{M}(\tilde{\tau}_1 \rightarrow \chi_i^0 \tau)|^2, \quad (11)$$

where

$$\begin{aligned} \mathcal{M}(\tilde{\tau}_1 \rightarrow \chi_i^0 (\bar{h}) \tau(h)) &= \bar{u}(\tau, h)(P_L a_{1i}^R + P_R a_{1i}^L)v(\chi_i^0, \bar{h}) \\ &= \frac{1}{2} [(a_{1i}^R + a_{1i}^L)H_s + (a_{1i}^L - a_{1i}^R)H_p]; \end{aligned} \quad (12a)$$

$$H_s = \sqrt{m_{\tilde{\tau}_1}^2 - (m_{\chi_i^0} + m_\tau)^2} \delta_{\lambda_f 0}, \quad (12b)$$

$$H_p = \sqrt{m_{\tilde{\tau}_1}^2 - (m_{\chi_i^0} - m_\tau)^2} \delta_{\lambda_f 0}(-)^{\bar{h}+1/2}. \quad (12c)$$

Here h and \bar{h} are helicities of χ_i^0 and τ , respectively, and $\lambda_f \equiv h - \bar{h}$. Recall that $m_{\chi_i^0}$ can have either sign here. The decay width $\Gamma(\tilde{\tau}_1 \rightarrow \nu_\tau \chi_i^-)$ can be obtained by replacing $a_{1i}^L \rightarrow b_{1i}$, $a_{1i}^R \rightarrow 0$, $m_{\chi_i^0} \rightarrow m_{\chi_i^-}$ and m_τ into zero. In this case $H_s = H_p(-)^{\bar{h}+1/2}$, thus $\tilde{\tau}_1$ decays only to a $\bar{h} = -1/2$ (left handed) neutrino state as it should be.

In the limit $m_\tau \ll m_{\tilde{\tau}_1}$, the difference between $|H_s|$ and $|H_p|$ is small. In such a case $\Gamma(\tilde{\tau}_1 \rightarrow \chi_i^0 \tau_R) \propto (a_{1i}^R)^2$ and $\Gamma(\chi_i^0 \tau_L) \propto (a_{1i}^L)^2$. The average polarization of the tau leptons is expressed in this limit as

$$P_\tau = \frac{Br(\chi_i^0 \tau_R) - Br(\chi_i^0 \tau_L)}{Br(\chi_i^0 \tau_R) + Br(\chi_i^0 \tau_L)} = \frac{(a_{1i}^R)^2 - (a_{1i}^L)^2}{(a_{1i}^R)^2 + (a_{1i}^L)^2}. \quad (13)$$

P_τ depends on both the stau mixing θ_τ and on the neutralino mixing N_{ij} . In the limit where χ_i^0 is gaugino-like, only N_{i1} and N_{i2} are sizable. This leads to $P_\tau = -1$ for $\cos \theta_\tau = 1$, while $P_\tau = +1$ for $\cos \theta_\tau = 0$ from Eq.(13) and (10). This results from the supersymmetric version of the chirality conservation of gauge interactions. On the other hand, if χ_i^0 is higgsino-like, one finds $P_\tau = +1$ for $\cos \theta_\tau = 1$ and $P_\tau = -1$ for $\cos \theta_\tau = 0$, because the supersymmetric version of Yukawa interactions flips the chirality of a (s)fermion. In section 2, we have discussed the measurement of θ_τ from the cross section for $\tilde{\tau}\tilde{\tau}^*$ production; even if $\tilde{\tau}_1$ is a mixed state of $\tilde{\tau}_L$ and $\tilde{\tau}_R$, one can still extract the information about neutralino mixing from the measured P_τ and θ_τ by solving Eq.(13).

To be more specific, the expression for P_τ for $\tilde{\tau}_R$ decay is

$$P_\tau(\tilde{\tau}_R \rightarrow \chi_1^0 \tau) = \frac{(\sqrt{2}N_{11} \tan \theta_W)^2 - \left(\frac{m_\tau}{\sqrt{2}m_W \cos \beta} N_{13}\right)^2}{(\sqrt{2}N_{11} \tan \theta_W)^2 + \left(\frac{m_\tau}{\sqrt{2}m_W \cos \beta} N_{13}\right)^2}. \quad (14)$$

In fig. 2 we show contours of fixed $P_\tau(\tilde{\tau}_R \rightarrow \chi_1^0 \tau)$ in the $M_1 - \mu$ plane for $\tan\beta = 10$ (fig. 2a) and $\tan\beta = 2$ (fig. 2b). P_τ decreases monotonically as M_1 increases for a fixed value of μ , as χ_1^0 is bino-like ($N_{11} \simeq 1$) for $M_1 \ll |\mu|$, while χ_1^0 is higgsino-like if $M_1 \gg |\mu|$ ($N_{11} \ll 1$). However, as the Yukawa coupling of τ is small ($\propto m_\tau/(m_W \cdot \cos\beta)$), $\tilde{\tau}_R$ still dominantly decays into τ_R after M_1 becomes larger than $|\mu|$. τ becomes unpolarized only when $M_1 \simeq 260$ GeV for $\mu = -100$ GeV and $\tan\beta = 10$. For $\tan\beta = 2$ and the same value of μ , P_τ becomes zero when $M_1 \simeq 500$ GeV due to the even smaller Yukawa coupling.

Although it is true that a gaugino-(higgsino)-like neutralino tends to give $P_\tau \simeq +1$ (-1), the actual value of P_τ depends strongly on $\tan\beta$. The polarization in the region where $|\mu| \ll M_1$ may be understood from the perturbative diagonalization of the neutralino mass matrix. Assuming that the differences between the diagonal elements is larger than the off-diagonal elements, the two higgsino-like neutralino masses and eigenvectors are given approximately by [16]

$$m_{\chi_H} = \mu(1 + \delta); \\ N_{Hi} = \frac{1}{\sqrt{2}} \left(\frac{M_Z \sin\theta_W (\cos\beta + \sin\beta)}{M_1 - \mu}, \frac{M_Z \cos\theta_W (\cos\beta + \sin\beta)}{\mu - M_2}, 1, -1 \right); \quad (15a)$$

$$m_{\chi_{H'}} = -\mu(1 + \delta'); \\ N_{H'i} = \frac{1}{\sqrt{2}} \left(\frac{M_Z \sin\theta_W (\cos\beta - \sin\beta)}{M_1 + \mu}, \frac{M_Z \cos\theta_W (\sin\beta - \cos\beta)}{M_2 + \mu}, 1, 1 \right); \quad (15b)$$

where

$$\delta = \frac{M_Z^2(1 + \sin 2\beta)}{2\mu} \left(\frac{\sin^2\theta_W}{\mu - M_1} + \frac{\cos^2\theta_W}{\mu - M_2} \right), \quad (16a)$$

$$\delta' = \frac{M_Z^2(1 - \sin 2\beta)}{2\mu} \left(\frac{\sin^2\theta_W}{\mu + M_1} + \frac{\cos^2\theta_W}{\mu + M_2} \right). \quad (16b)$$

For $|M_1| \gg |\mu|$, one finds $\chi_1 = \chi_{H'}$, $\chi_2 = \chi_H$ for $\mu < 0$, while $\chi_1 = \chi_H$ and $\chi_2 = \chi_{H'}$ for $\mu > 0$.

Inserting Eq.(15) into Eq.(14), we estimate that $P_\tau(\tilde{\tau}_R \rightarrow \chi_1 \tau) = 0$ at $M_1 \simeq 280$ GeV for $\mu = -100$ GeV and $\tan\beta = 10$, which roughly agrees with the numerical result shown in the figure. For $\tan\beta = 2$, the value of M_1 that leads to $P_\tau = 0$ is estimated to be about 550 GeV.

Fig. 2b shows quite asymmetric behavior of the τ polarization for the sign of μ . This is due to the fact that the bino component of χ_1^0 is enhanced (suppressed) by the factor $\cos\beta \pm \sin\beta$ for $\mu > 0$ ($\mu < 0$). For large $\tan\beta$, this becomes irrelevant as $\cos\beta \ll \sin\beta$.

3b) The branching ratio into the lightest neutralino

As discussed in section 2, the decay mode $\tilde{\tau}_1 \rightarrow \tau \chi_1^0$ gives the simplest signal for $\tilde{\tau}_1$ production, while the modes $\tilde{\tau}_1 \rightarrow \nu \chi_i^-$, $\tau \chi_j^0 (j > 2)$ are more challenging. In this subsection, we are going to see how big the branching ratio into the lightest neutralino is among the other branching ratios.

In fig. 3, we show the M_1 dependence of the branching ratios of $\tilde{\tau}_1$, for $\mu = -300$ GeV and $m_{\tilde{\tau}_1} = 150$ GeV. In the region of parameter space shown in this figure, $\chi_1^0 \simeq \tilde{B}$, $\chi_2^0 \simeq \tilde{W}_3$ and $\chi_1^- \simeq \tilde{W}$, and only the decays into those -ino's are kinematically accessible ($m_{\chi_2^0} \simeq m_{\chi_1^-} \simeq M_2$, $m_{\chi_1^0} \simeq M_1$). The decays into χ_2^0 and χ_1^+ are closed when $M_1 \gtrsim 75$ GeV.

As discussed above, the decay of $\tilde{\tau}_1$ into $\tau_{R(L)}$ has 100% branching ratio for $\tilde{\tau}_1 = \tilde{\tau}_{R(L)}$. $Br(\tilde{\tau}_1 \rightarrow \chi_1^0 \tau)$ is however quite small if $\tilde{\tau}_1 \simeq \tilde{\tau}_L$ as long as the other decay modes are kinematically accessible, less than 20% for $M_2 \ll m_{\tilde{\tau}_1}$. This can be explained in the limit $m_{\tilde{\tau}_1} \gg M_2$ as the ratios of couplings:

$$\begin{aligned} \Gamma(\tilde{\tau}_1 \rightarrow \chi_1^0 \tau) : \Gamma(\tilde{\tau}_1 \rightarrow \chi_1^0 \tau) : \Gamma(\tilde{\tau}_1 \rightarrow \chi_1^- \tau) \\ \text{for } \tilde{\tau}_L = (a_{L1}^L)^2 : (a_{L2}^L)^2 : (b_{L1})^2 = \left(\frac{\tan \theta_W}{\sqrt{2}} \right)^2 : \frac{1}{2} : 1, \\ \text{for } \tilde{\tau}_R = (a_{R1}^L)^2 : (a_{R2}^L)^2 : 0 = 1 : 0 : 0. \end{aligned} \quad (17)$$

From Eq.(17), one can see that $\sum_i Br(\chi_i^0 \tau)$ is less than 50% and $Br(\chi_1^0) \sim Br(\chi_2^0 \tau) \tan^2 \theta_W$ for $\tilde{\tau}_L$. If $\tilde{\tau}_1$ is $\tilde{\tau}_R$, the modes into χ_2^0 or χ_1^- have zero branching ratio, as $\tilde{\tau}_R$ is an SU(2) singlet.

In fig. 3, we also show the branching ratios into $\chi_1^0 \tau_R$ and $\chi_1^0 \tau_L$ for $\sin^2 \theta_\tau = 0.7$ (dotted lines). For $M_1 < 75$ GeV, the branching ratios into χ_2^0 or χ_1^- rise rather quickly reducing the ratio into χ_1^0 at the same time. This is again because the SU(2) gauge coupling is larger than the U(1) coupling. On the other hand, the branching ratio into $\chi_1^0 \tau_R$ is still more than 80% of the total $\chi_1^0 \tau$ decay. Indeed, $P_\tau(\tilde{\tau}_1 \rightarrow \chi_1^0 \tau)$ for $\chi_1^0 \simeq \tilde{B}$ is

$$P_\tau = \frac{(\sin \theta_\tau a_{R1}^R)^2 - (\cos \theta_\tau a_{L1}^L)^2}{(\sin \theta_\tau a_{R1}^R)^2 + (\cos \theta_\tau a_{L1}^L)^2} = \frac{4 \sin^2 \theta_\tau - \cos^2 \theta_\tau}{4 \sin^2 \theta_\tau + \cos^2 \theta_\tau}. \quad (18)$$

The factor of 4 comes from the fact that τ_R has twice as large a hypercharge as τ_L . Due to this factor, if $\sin \theta_\tau$ is close to one, P_τ has very mild dependence on θ_τ , while P_τ is quite sensitive to the $\tilde{\tau}_R$ component when $\tilde{\tau}_1$ is close to $\tilde{\tau}_L$.

So far we have been discussing the case where $M_1 < m_{\tilde{\tau}_1} \ll |\mu|$. How about the case $|\mu| < m_{\tilde{\tau}_1}$? In fig. 4a and fig. 4b, we show the branching ratios of $\tilde{\tau}_R$ and $\tilde{\tau}_L$ for $\mu = -100$ GeV, $m_{\tilde{\tau}_1} = 150$ GeV and $\tan \beta = 10$, varying M_1 from 50 GeV to 500 GeV. For $M_1 \gg 100$

GeV, χ_1^0 , χ_2^0 and χ_1^- are higgsino-like, and the decays into them are kinematically accessible, as $m_{\chi_1^0}$, $m_{\chi_2^0}$ and $m_{\chi_1^-} \simeq |\mu|$. On the other hand, for $M_1 < 100$ GeV, χ_1^0 has a large \tilde{B} component. However, due to the relatively small value of $|\mu|$, all χ_i 's have non-negligible mixing between gauginos and higgsinos, if M_1 is small.

In fig. 4a we show branching ratios of $\tilde{\tau}_R$ decays. Solid lines show branching ratios into the lightest neutralino. The solid line which decreases as M_1 increases is for $Br(\chi_1^0\tau_R)$, while the one which increases as M_1 increases is for $Br(\chi_1^0\tau_L)$. Dashed lines are for the sum of the branching ratios into neutralinos, and the dot-dashed line is for $Br(\chi_1^-\nu_\tau)$. Branching ratios into the other final states are negligible, and we did not show them on the plot.

For $M_1 < 100$ GeV, the bino component of χ_1^0 decreases as M_1 increases, while increasing the bino component of heavier states. Thus $Br(\chi_1^0\tau_R)$ decreases, keeping $\sum_i Br(\chi_i^0\tau_R)$ about constant. Increasing M_1 even more results in the kinematical suppression of the decay into $\chi_3^0 \simeq \tilde{B}$, reducing $\sum_i Br(\chi_i^0\tau_R)$ while increasing $Br(\chi_1^0\tau_R)$. However, for $M_1 > 150$ GeV the branching ratios into τ_R decrease monotonically, as the bino components of χ_1^0 and χ_2^0 become smaller. Finally, in the limit where $M_1 \gg |\mu|$, only decays into $\chi_i^0\tau_L$ and $\chi_i^-\nu_\tau$ survive. $Br(\chi_1^0\tau_L) \simeq Br(\chi_2^0\tau_L) \simeq 1/2 Br(\chi_1^-\nu_\tau)$ and $Br(\chi_1^0\tau_L) \simeq 25\%$ in this limit.

The branching ratios for $\tilde{\tau}_L$ decay for the same value of μ and $\tan\beta$, which are shown in fig. 4b, are quite different from those of $\tilde{\tau}_R$. For $M_1 < |\mu|$, $Br(\chi_1^0\tau_L)$ (solid line) is very small, while $Br(\chi_i^0\tau_L)$ (dashed line) is larger than that of $Br(\chi_i^-\nu_\tau)$ (dot-dashed line). The branching ratios are also quite different from those for $\mu = -300$ GeV shown in fig. 2. This is due to a cancellation between N_{11} and N_{12} in a_{L1}^L (see Eq.(10)), which determines $Br(\chi_1^0\tau)$; the wino component N_{12} is smaller than the bino component N_{11} and has the opposite sign, while the gauge coupling is larger for the wino component, making the total a_{L1}^L very small. For $M_1 \simeq 140$ GeV, these two components cancel exactly, leading to zero branching ratio into $\chi_1^0\tau_L$. On the other hand, N_{21} and N_{22} have the same sign in this region, giving a large a_{L2}^L coupling, and hence a large $Br(\chi_2^0\tau_L)$.

$Br(\chi_1^0\tau_L)$ reduces very quickly once $|M_1| > 100$ GeV. This is because the decay into χ_3^0 becomes kinematically suppressed, and furthermore not only a_{L1}^L , but also a_{L2}^L tends to be suppressed due to a cancellation between the wino component and the bino component. The cancellation also can be seen from Eq.(15), where the wino like and the bino like components of the eigenvectors have opposite sign if $M_1, M_2 \gg |\mu|$. As a result, the branching ratios into τ_R rise rather quickly. Notice also that $Br(\chi_i^-\nu)$ is very small in this region, as $\tilde{\tau}_L$ does not couple to charged Higgsino. Therefore $Br(\chi_1^0\tau_R) \simeq Br(\chi_2^0\tau_R) \simeq 50\%$ in the limit $M_1 \gg |\mu|$.

There are several generic features which have been illustrated in fig. 3 and 4. First, if $M_1 \ll \mu$, χ_1^0 is bino-like and the branching ratio to χ_1^0 is dominant for $\tilde{\tau}_R$, while it is rather small for $\tilde{\tau}_L$ as long as wino-like χ_2^0 and χ_1^- modes are accessible (see fig. 3). On the other

hand, if the lightest neutralino is higgsino-like ($|\mu| \ll M_1$) and $m_{\tilde{\tau}_1} < M_1$ (which implies that decay modes into gaugino-like -inos are kinematically forbidden), there is always a substantial branching ratio into χ_1^0 , more than 25% (50%) for $\tilde{\tau}_R$ ($\tilde{\tau}_L$) (fig. 4). It should be noted, however, that the mass differences between χ_1^0 , χ_2^0 and χ_1^- are very small in this limit. For the parameters shown in fig. 4, $|m_{\chi_1^0}| = 94.1$ GeV, $|m_{\tilde{\chi}_2^0}| = 106.6$ GeV and $|m_{\chi_1^-}| = 100.6$ GeV for $M_1 = 400$ GeV. For such a small mass difference, the decays of χ_2^0 and χ_1^- give very soft hadron jets or leptons, which may not be tagged. Thus it is possible that we measure the sum of the decay modes $\tilde{\tau}_1 \rightarrow \chi_i^0 \tau$ altogether and $\tilde{\tau}_1 \rightarrow \chi_i^- \nu$ is not detectable at all. However, the relation between the parameters of the neutralino sector ($M_1, M_2, \mu, \tan\beta$) and P_τ is roughly unchanged, even if we do the inclusive analysis. Finally, if $|\mu| \simeq M_1$, the neutralinos and charginos are strong mixtures of gauginos and higgsinos. $\tilde{\tau}_R$ decays into $\chi_i^0 \tau$ through their bino component, while $\tilde{\tau}_L$ decays into both $\chi_i^0 \tau$ and $\chi_i^- \nu$ through their bino and wino components. Interferences between gaugino components can be very important for $\tilde{\tau}_L$. $\tilde{\tau}_{R(L)}$ decays into $\tau_{R(L)}$, since the higgsino-(s)fermion coupling is much smaller than the gaugino coupling unless $\tan\beta \gg 1$.

We should mention that production of $\tilde{\nu}_\tau$ may not be distinguishable from $\tilde{\tau}_1$ production. $\tilde{\nu}_\tau$ production occurs through s channel Z^0 boson exchange. The production cross section in the limit $s \gg m_Z$ can be obtained by replacing $1 + \tan^2 \theta_W \rightarrow -1 + \tan^2 \theta_W$ and taking $\sin \theta_\tau = 0$ and $\cos \theta_\tau = 1$ in Eq.(5); if $P_e = 1$, the cross section of $\tilde{\nu}_\tau$ production is equal to that for $\tilde{\tau}_L$. The $\tilde{\nu}_\tau$ then decays into $\nu_\tau \chi_i^0$ or $\tau \chi_i^+$, and the decay into χ_i^+ gives $\tau + (2 \text{ jets or 1 lepton}) + \text{missing momentum}$. Then the event signature is very close to that of $\tilde{\tau}_1 \rightarrow \tau \chi_i^0 (i \geq 2)$ if the χ_i^+ decays hadronically. Notice that $\tilde{\nu}_\tau$ production always associates with $\tilde{\tau}_1$ production if the $\tilde{\tau}_1$ is close to $\tilde{\tau}_L$. This is because the sneutrino soft breaking mass parameter is common to that of $\tilde{\tau}_L$ due to $SU(2) \times U(1)$ symmetry; $m_{\tilde{\tau}_L} \simeq m_{\tilde{\nu}_\tau}$ up to small D term effects. However, the polarization of the τ lepton from $\tilde{\nu}_\tau$ decay is left-handed (right-handed) for gaugino-(higgsino-)like χ_1^- , thus again the relation between P_τ and $M_1, M_2, \mu, \tan\beta$ is retained.[†]

4) Measurement of $P_\tau(\tilde{\tau}_1 \rightarrow \tau \chi_1^0)$

In this section, we discuss how to measure $P_\tau(\tilde{\tau}_1 \rightarrow \tau \chi_1^0)$. We have already mentioned in the beginning of section 3 that the measurement can be done through the energy distribution of the decay products of the τ lepton. However, difficulties arise as the τ lepton does not have a

[†]As will be discussed in sec. 4, P_τ can be measured only through the energy distribution of the decay products of the τ lepton. In order to be able to claim that $\tilde{\nu} \rightarrow \tau \chi_i^+$ decay does not disturb P_τ information from $\tilde{\tau} \rightarrow \tau \chi_j^0$ decay, $m_{\chi_i^+} \simeq m_{\chi_i^0}$ should be satisfied. This is actually the case if $M_1 \gg |\mu|$ or $|\mu| \gg M_2$, as a wino-like neutralino and a wino-like chargino and also higgsino-like neutralinos and a higgsino-like chargino are very close in mass.

fixed energy; thus the energy distribution of the decay products depends not only P_τ but also on $m_{\tilde{\tau}_1}$ and $m_{\chi_1^0}$.

The energy distribution of the τ lepton from the decay $\tilde{\tau}_1 \rightarrow \tau \chi_1^0$ is flat between E_τ^{\max} and E_τ^{\min} , where

$$E_\tau^{\max(\min)} = \frac{E_\tau^c \pm p_\tau^c v_{\tilde{\tau}_1}}{\sqrt{1 - v_{\tilde{\tau}_1}^2}}. \quad (19)$$

Here E_τ^c and p_τ^c are the energy and the momentum of the τ lepton in the rest frame of $\tilde{\tau}_1$, and $v_{\tilde{\tau}_1}$ is the velocity of $\tilde{\tau}_1$ in the laboratory frame:

$$\begin{aligned} E_\tau^c &= \frac{m_{\tilde{\tau}_1}^2 + m_\tau^2 - m_{\chi_1^0}^2}{2m_{\tilde{\tau}_1}}, \\ p_\tau^c &= \sqrt{E_\tau^{c2} - m_\tau^2}, \\ v_{\tilde{\tau}_1} &= \sqrt{1 - (2m_{\tilde{\tau}_1}/\sqrt{s})^2}. \end{aligned} \quad (20)$$

For \tilde{e} and $\tilde{\mu}$ production, the end points of the energy distribution of the final state lepton can be used to determine $m_{\chi_1^0}$ and m_l [14]. For the case of $\tilde{\tau}$, however, the τ lepton further decays into $A\nu$, where $A = e\nu, \mu\nu, \pi, \rho, a_1, \dots$. In the limit $E_\tau \gg m_\tau$, A keeps the original direction of the momentum of the τ lepton; however, the energy of A is softer as some of the τ energy is carried away by ν_τ .

As we have already mentioned, the energy distribution of the final state particles strongly depends on P_τ . For illustrative purposes we discuss the decay mode $\tau \rightarrow \pi \nu_\tau$ first. The energy distribution of the π^- arising from polarized τ leptons is very simple:

$$\frac{1}{\Gamma_\tau} \frac{d\Gamma}{dz} (\tau^\pm \rightarrow \pi^\pm \nu_\tau) = B_\pi [1 \mp P_\tau(2z - 1)], \quad (21)$$

where $z \equiv E_\pi/E_\tau$ and $B_\pi \simeq 0.12$ is the branching ratio of τ into π . One can see that the energy distribution is harder for τ_R^- than for τ_L^- . The energy distribution of the π coming from the cascade decay of $\tilde{\tau}_1$ is then given by:

$$\begin{aligned} &\frac{1}{\Gamma_{\tilde{\tau}_1}} \frac{d\Gamma}{dy} (e^+ e^- \rightarrow \tilde{\tau}_1 \tilde{\tau}_1^* \rightarrow \chi_1^0 \tau^- \rightarrow \pi^- \nu_\tau) \\ &= \mathcal{F} \int_{\max(x^{\min}, y)}^{x^{\max}} \frac{1}{x} \cdot \left[1 + P_\tau \left(\frac{2y}{x} - 1 \right) \right] \\ &= \begin{cases} \mathcal{F} \left[(1 - P_\tau) \log \left(\frac{x^{\max}}{y} \right) + 2P_\tau \left(1 - \frac{y}{x^{\max}} \right) \right] & \text{for } y > x^{\min}, \\ \mathcal{F} \left[(1 - P_\tau) \log \left(\frac{x^{\max}}{x^{\min}} \right) + 2yP_\tau \left(\frac{1}{x^{\min}} - \frac{1}{x^{\max}} \right) \right] & \text{for } y < x^{\min}, \end{cases} \end{aligned} \quad (22)$$

where $y = E_\pi/E_{\tilde{\tau}_1} = 2E_\pi/\sqrt{s}$, $x^{\min} = 2E_\tau^{\min}/\sqrt{s}$, $x^{\max} = 2E_\tau^{\max}/\sqrt{s}$ and

$$\mathcal{F} = \frac{1}{2v_{\tilde{\tau}_1} p_\tau^c} \sqrt{1 - v_{\tilde{\tau}_1}^2} B_\pi B_{\chi_1^0 \tau}. \quad (23)$$

In fig.5 we show the energy distribution of the π for $P_\tau = \pm 1$ and 0, where $m_{\tilde{\tau}} = 150$ GeV, $m_{\chi_1^0} = 100$ GeV and $\sqrt{s} = 500$ GeV. For $P_\tau = +1$, the distribution is peaked at $y = x^{\min} \simeq 0.06$ and goes to zero at $y = 0$. On the other hand, the distribution is strongly peaked at $y = 0$ for $P_\tau = -1$. 48% of the total events has $y < x^{\min}$ for $P_\tau = -1$, while it is only 10% for $P_\tau = +1$.

Given the relatively hard E_π spectrum peaked at x^{\min} , this mode might be suitable to measure x^{\min} and x^{\max} if $P_\tau \simeq 1$. If x^{\min} and x^{\max} can be measured, it is easy to determine P_τ and $\sigma(e^+e^- \rightarrow \tilde{\tau}_1 \tilde{\tau}_1^*) \cdot Br(\tilde{\tau}_1 \rightarrow \tau \chi_1^0)$. For example, one may count the number of events with $y > x^{\min}$ and, say, $y > (x^{\min} + x^{\max})/2$, and carry out a two parameter fit of P_τ and $\sigma \cdot Br$. For the parameters used in fig. 5, 52% (90%) of the events are in $y > x^{\min}$ for $P_\tau = -1$ (+1), while 4% (22%) of the events go above $(x^{\min} + x^{\max})/2$. For $\mathcal{L} = 100 fb^{-1}$, $\sigma(e^+e^- \rightarrow \tilde{\tau}_1 \tilde{\tau}_1^*) = 0.1$ pb and $Br(\tilde{\tau}_1 \rightarrow \chi_1^0 \tau) = 1$, more than one thousand π^- events are available for this analysis before any selection cuts have been applied.

For the above analysis, we needed to know x^{\max} to determine ‘high’ and ‘low’ energy samples. The precise measurement of this quantity turns out to be difficult for $P_\tau \simeq -1$ due to the very small probability of π to get an energy near the kinematical maximum; for $m_{\tilde{\tau}_1} = 150$ GeV and $m_{\tilde{\tau}_1} = 100$ GeV, only 1% of the events have energy above $y > (x^{\min} + 2x^{\max})/3$ ($E_\pi > 87$ GeV). Furthermore, if x^{\min} is small, its measurement will also be difficult. This is due to the huge background from $\gamma\gamma \rightarrow \tau\bar{\tau}, q\bar{q}$ in the small y region. The total cross section for the process $\gamma\gamma \rightarrow \tau\bar{\tau}$ is about 7 pb.[‡] Imposing a minimum visible energy cut of 50 GeV, in addition to other selection cuts, the background can be reduced to less than 1 fb[6]. The visible energy cut might be weakened, but still it would be very difficult to measure the shape of the energy distribution over the $\gamma\gamma$ background in the small y region. Notice that the detection efficiency of the π depends strongly on P_τ and E_{vis} . Together with the uncertainty of x^{\max} , this makes the determination of P_τ and $\sigma(e^+e^- \rightarrow \tilde{\tau}_1 \tilde{\tau}_1^*) \cdot Br(\chi_1^0 \tau)$ quite difficult if one uses this τ decay mode only. The problem of measuring x^{\max} arises also in the decay mode $\tau \rightarrow \mu(e)$, as $\mu(e)$ from τ decay has a very soft energy spectrum both for $P_\tau = \pm 1$.

Decay modes into heavier mesons, which give multiple pions in the final states, are more suitable to measure $x^{\max(\min)}$. The $\tau \rightarrow \rho\nu$ mode has a branching ratio of about 23%, and the $\tau \rightarrow a_1\nu$ mode has a branching ratio of about 15%. $\tau_{L(R)}$ decays dominantly into the longitudinal (transverse) component of ρ or a_1 , and they tend to get most of the τ energy [10]. The ρ (a_1) further decays into 2π (3π) states. Thus even when $P_\tau = -1$, one might be able to measure $x^{\max(\min)}$ by studying the shape of energy distributions of these multiple pion systems.

[‡]There would also be a background from beamstrahlung, which strongly depends on the machine design.

Even if x^{\min} is not measurable due to backgrounds from $\gamma\gamma$ production, one might still be able to determine $m_{\chi_1^0}$ from the production of other sparticles. As we mentioned earlier, we can measure $m_{\chi_1^0}$ through the measurement of the end points of the energy distribution of $e(\mu)$ from $\tilde{e}(\tilde{\mu})$ (if it is produced at the same beam energy). Another possibility is to perform a threshold scan to determine $m_{\tilde{\tau}_1}$. Combining the information of one of the two masses with x^{\max} , one can extract the other mass, and thus x^{\min} . Once $x^{\min,\max}$ are known, P_τ can be extracted from the energy distribution of π as described earlier. P_τ also can be measured independently by studying the distributions of the difference of the energy between decay pions from ρ and a_1 . Namely, transversely polarized ρ favors equal splitting of the ρ energy between the two decay pions, whereas longitudinally polarized ρ leads to a large difference of the π^- and π^0 energies. For a_{1T} , all three pions have a tendency to share equally the energy of a_1 . On the other hand, a_{1L} again favors configurations in which one or two of the pions are soft. Notice that, unlike the case of the decay mode $\tau \rightarrow \pi\nu$, the dependence of the detection efficiency on P_τ and E_{vis} is not severe for these modes, as P_τ mainly affects the polarization of the vector mesons, rather than its energy distribution. A detailed discussion of the energy distribution may be found in [10].

There are several sources of background other than W^+W^- boson production which has been discussed earlier. The cross section for Z^0Z^0 production is about 0.15 pb for $|\cos\theta| < 0.8$ and a non-polarized electron beam [12]. Requiring one Z^0 boson to decay into neutrinos and the other into a tau pair reduces this background to about 1.8 fb. The polarized electron beam and a cut on the angle between the decay products of the two tau leptons would make the background even smaller. $\sigma(e^+e^- \rightarrow Z^0h)$ is about 60 fb (for $m_h \sim 120$ GeV)[12]. Demanding that the Z boson decays into neutrinos and the Higgs boson goes into a tau pair reduces this to about 1.8 fb. These backgrounds, including those coming from W boson production, can be calculated and subtracted reliably from the signal events. Notice that the polarization of τ leptons is zero for those coming from Higgs bosons, and -1 for those coming from W bosons. Those polarizations affect the energy distribution of its decay products, and have to be taken into account for the estimation of the energy distribution of the background. We finally remark that initial and final state radiation change the energy distribution of the signal, which has not been included in this paper. MC simulations both of signals and backgrounds are in progress[17].

5. Discussion and Summary

What do we learn from the study of production and decay of $\tilde{\tau}_1$? If $\tilde{\tau}_1$ is the second lightest SUSY particle, the branching ratio into $\tilde{\tau}_1 \rightarrow \chi_1^0\tau$ is 100%. In this case, we can measure $m_{\tilde{\tau}_1}$,

$m_{\chi_1^0}$, $\sigma(e^+e^- \rightarrow \tau\bar{\tau})$ and P_τ from the study of $\tilde{\tau}_1$ production and decay. From the production cross section and $m_{\tilde{\tau}_1}$, one can extract θ_τ as discussed in section 2. The determination of θ_τ is very important as A_τ appears only in the left-right mixing of $\tilde{\tau}_1$ or the $\tilde{\tau}_L\tilde{\tau}_R H_1$ coupling. Once θ_τ is known, the measurement of P_τ would give one constraint on neutralino mixing. Together with the measured value of $m_{\chi_1^0}$, we get two constraints on the four parameters in the neutralino and the chargino sector ($M_1, M_2, \mu, \tan\beta$).

On the other hand, if the $\tilde{\tau}_1$ is not the second lightest SUSY particle, we have to measure the branching ratios to other neutralinos and charginos to determine the total $\tilde{\tau}_1$ pair cross section. As we have discussed in section 3, the $Br(\chi_1^0\tau)$ might be small if $\tilde{\tau}_1 \simeq \tilde{\tau}_L$ and decay modes into heavier -inos are kinematically accessible. The ratio $Br(\chi_1^0\tau) / Br(\chi_i^0\tau)$ can be studied by taking the ratio $N(\tau\bar{\tau} + \text{missing momentum}) / N(\tau\bar{\tau} + \text{missing momentum} + \text{jets or leptons})$. The branching ratio into $\tilde{\tau}_1 \rightarrow \chi_j^- \nu_\tau$ would be more difficult to measure precisely, as one has to take the ratio $N(\tau + \text{missing momentum} + \text{jets or a lepton}) / N(\tau\bar{\tau} + \text{missing momentum} + \text{jets or leptons})$. Because only one τ lepton is required for the process where one $\tilde{\tau}_1$ decays into a chargino, the background from W^+W^- production is bigger, about 5 fb even with $P_e = 1$. On the other hand, in such a scenario pairs of charginos and heavier neutralinos can be produced directly at the same collider. The study of these events, together with the measured decay branching ratios of $\tilde{\tau}_1$, should give us much information on the neutralino and chargino sector.

For the case with several SUSY particles which are close in mass to $\tilde{\tau}_1$ or χ_1^0 , the analysis would be considerably more difficult. Even in such a case, P_τ gives a constraint to the combination of θ_τ and the mixing matrix of the neutralinos (N_{ij}).

Several comments are in order. b (\tilde{b}) and t (\tilde{t}) also have non-negligible Yukawa couplings to higgsinos, thus the polarization of bottom and top quarks from \tilde{b} and \tilde{t} decays depends on both squark and neutralino mixing. However, the bottom quark quickly forms a hadronic bound state, losing the information of the original polarization. A top quark decays before making a bound state. However, due to the large top mass suggested by CDF and D0[18], at $\sqrt{s} = 500$ GeV the produced top quark is too slow to allow a reliable measurement of its polarization. Thus $\tilde{\tau}_1$ decay is the only place where one can possibly measure the polarization of the final state fermion.

In this paper we discussed the production of $\tilde{\tau}_1$ at the proposed linear colliders with $\sqrt{s} = 500$ GeV. The same analysis in principle can be done for e^+e^- colliders with smaller beam energy. At LEPII, however, W^+W^- pairs are produced close to threshold, which makes precision studies of the production of SUSY particles rather difficult. Besides, the integrated luminosity is expected to be of order a few hundred pb^{-1} . Given that the production cross section of $\tilde{\tau}_1\tilde{\tau}_1^*$ is about 0.3 pb for $m_{\tilde{\tau}_R} = 60$ GeV and $\sqrt{s} = 170$ GeV, a measurement the polarization of τ leptons produced in $\tilde{\tau}_1$ decays seems to be very challenging.

Finally, we would like to discuss the mass spectrum of superparticles $(\tilde{\tau}_1, \chi_i^0, \chi_i^-)$ in the minimal supergravity model, as this model has been investigated in many articles recently. In this model, the mass spectrum at the weak scale is expressed only by the 4 parameters m, M, A, B at the GUT scale; m is a universal scalar mass, and M a universal gaugino mass. The relations to the weak scale quantities m_R^2, m_L^2, M_1 and M_2 are given by RG running from the GUT scale. The common mass parameters at the GUT scale leads to equal soft slepton masses m_R^2, m_L^2 for \tilde{l}_R and \tilde{l}_L of all generations if the lepton Yukawa couplings are negligible; numerically one has [11]

$$m_R^2 = m^2 + 0.87M_1^2, \quad (24a)$$

$$m_L^2 = m^2 + 3M_1^2, \quad (24b)$$

$$M_1 = \frac{5}{3} \tan^2 \theta_W M_2 = 0.42M. \quad (24c)$$

In particular, the relation $m_{\tilde{\tau}_R} < m_{\tilde{\tau}_L}$ follows from Eqs.(24), suggesting that $\tilde{\tau}_1 \simeq \tilde{\tau}_R$. The relation between M_1 and $m_{\tilde{\tau}_R}$ is more complicated. If m and the D -term are negligible compared to M_1 , $m_{\tilde{\tau}_R}$ could be lighter than M_1 , as the coefficient of M_1^2 in Eq.(24a) is smaller than 1. In such a case, the decay modes into gaugino-like states are closed.

The masses $m_{R(L)}$ are common to all sleptons if the τ Yukawa coupling is negligible. However, $\tilde{\tau}_1$ can be lighter than the other sleptons due to two effects. If $\tan\beta$ is big, the Yukawa coupling of τ is not negligible so that $m_{\tilde{\tau}_{L,R}}$ at the weak scale get extra negative contributions from the Yukawa interaction in the RG running from the GUT scale, the effect being larger for $m_{\tilde{\tau}_R}$. Moreover, there can be substantial mixing between $\tilde{\tau}_L$ and $\tilde{\tau}_R$. Notice that this mixing tends to increase with increasing $\tan\beta$ due to the term $\mu \cdot \tan\beta \cdot m_\tau$ in the off-diagonal element of the mass matrix of Eq.(1)[11]. Mixing reduces $m_{\tilde{\tau}_1}$ even further, so that $m_{\tilde{\tau}_1} < m_{\tilde{\tau}_R} < m_{\tilde{e}_R} \simeq m_{\tilde{\mu}_R}$ in minimal supergravity models with $\tan\beta \gg 1$. If in addition $m_{\tilde{\tau}_1} < m_{\chi_1^-}$, $\tilde{\tau}_1$ might be the only charged sparticle that is detectable at a given collider.

It is often said that the minimal supergravity model with radiative symmetry breaking predicts $|\mu| > M_1$ for heavy top. This is due to the fact that the Higgs mass parameters at the weak scale should not be too negative, since otherwise the Higgs potential would be unbounded from below. One of these mass parameters gets a large negative radiative correction coming from (s)top loops when going down from the GUT scale. As the squared Higgs mass at the GUT scale is the sum of squares of the soft mass parameter m and the higgsino mass parameter μ , the large top Yukawa correction requires large $|\mu|$ [11, 19]. In particular, for $m_t > 160$ GeV or so, the lightest neutralino is always bino-like [16]; thus $m_{\tilde{\tau}_1}$ should be larger than the bino-like neutralino mass $\simeq M_1$, to ensure that the lightest SUSY particle is neutral. One would then expect $\tilde{\tau}_1$ to dominantly decay into gauginos. However, this constraint would be drastically

weakened if one allows unequal masses for Higgs bosons already at the GUT scale. Recently such models have been constructed explicitly [20]. Thus it is possible to find cases where $\tilde{\tau}_1$ dominantly decays into higgsino-like neutralinos and charginos even in a certain class of supergravity GUT models.

In summary, in this paper we proposed the measurement of the polarization of τ lepton that arises from the decay $\tilde{\tau}_1 \rightarrow \chi_1^0 \tau$. If $\tilde{\tau}_1$ decays dominantly into $\chi_1^0 \tau$, one can measure $m_{\tilde{\tau}_1}$, $m_{\chi_1^0}$, and θ_τ , and get another constraint on the higgsino and gaugino mixing of neutralinos by studying $\tilde{\tau}_1$ decay only. We would like to emphasize that, if the production of other SUSY particles is also observed at future linear colliders or hadron colliders, we could determine or even over-constrain all SUSY parameters in the neutralino-chargino sector and the matter sector. This would allow to simultaneously establish the existence of supersymmetry and specify the GUT scale model that reproduces the observed mass spectrum at the weak scale.

Acknowledgment

I thank to K. Hagiwara, K. Fujii, T. Tsukamoto, Y. Okada, M. Drees and K. Hikasa for helpful discussions, comments, and encouragements. I would also like to thank M. Drees for reading the manuscript carefully. This work is supported in part by the Grant-in-aid for Scientific Research from the Ministry of Education, Science and Culture of Japan (06740236).

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Figure Captions

Fig.1 The cross section for $\tilde{\tau}_1\tilde{\tau}_1^*$ production at an e^+e^- collider as a function of the scalar tau mixing angle θ_τ . We have taken $m_{\tilde{\tau}_1} = 150$ GeV and $\sqrt{s} = 500$ GeV. For an unpolarized electron beam ($P_{e^-} = 0$), the cross section depends very mildly on θ_τ .

Fig.2 Contours of constant polarization of the τ lepton arising from the decay $\tilde{\tau}_R \rightarrow \tau\chi_1^0$ in the plane spanned by M_1 and μ . We have assumed the τ lepton to be relativistic. The regions excluded by LEP experiments are also shown in the figure. P_τ decreases as M_1 increases for fixed value of μ .

Fig.3 Branching ratios of the lighter scalar tau into a neutralino (χ_i^0) plus a τ or a chargino (χ_i^-) plus a ν_τ , as a function of M_1 for fixed $m_{\tilde{\tau}_1} = 150$ GeV, $\mu = -300$ GeV, $\tan\beta = 10$ and $\sqrt{s} = 500$ GeV; $\chi_1^0 \simeq \tilde{B}$, $\chi_2^0 \simeq \tilde{W}^3$ and $\chi_1^- \simeq \tilde{W}$ for the parameters shown in this figure. If $\tilde{\tau}_1$ is $\tilde{\tau}_R$, $\tilde{\tau}_1$ decays into $\chi_1^0\tau_R$ dominantly (the upper solid line), while $\tilde{\tau}_L$ decays into $\chi_1^0\tau$, $\chi_2^0\tau$ and $\chi_1^-\nu_\tau$ as long as these decay modes are kinematically accessible.

Fig.4 a) Branching ratios of $\tilde{\tau}_R$ into $\chi_i^0\tau$ and $\chi_i^-\nu_\tau$ for $m_{\tilde{\tau}_R} = 150$ GeV, $\mu = -100$ GeV and $\sqrt{s} = 500$ GeV, varying M_1 from 50 GeV to 500 GeV. For $M_1 \gg |\mu|$, χ_1^0 , χ_2^0 and χ_1^- are dominantly higgsinos.

b) Branching ratios of $\tilde{\tau}_L$ into $\chi_i^0\tau$ and $\chi_i^-\nu_\tau$ for the same parameters as in Fig. 4a.

Fig.5 Energy distribution of π from the $\tilde{\tau}$ cascade decay $\tilde{\tau} \rightarrow \chi_i^0\tau$ and $\tau \rightarrow \pi\nu_\tau$ at a e^+e^- collider; $\sqrt{s} = 500$ GeV, $m_{\tilde{\tau}} = 150$ GeV and $m_{\chi_1^0} = 100$ GeV. The three curves are for different τ polarization $P_\tau = 1, 0, -1$.

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